Disclosure to Promote the Right To Information

Whereas the Parliament of India has set out to provide a practical regime of right to information for citizens to secure access to information under the control of public authorities, in order to promote transparency and accountability in the working of every public authority, and whereas the attached publication of the Bureau of Indian Standards is of particular interest to the public, particularly disadvantaged communities and those engaged in the pursuit of education and knowledge, the attached public safety standard is made available to promote the timely dissemination of this information in an accurate manner to the public.

"जानने का अधिकार, जीने का अधिकार"
Mazdoor Kisan Shakti Sangathan
"The Right to Information, The Right to Live"

"पुराने को छोड़ नये के तरफ"
Jawaharlal Nehru
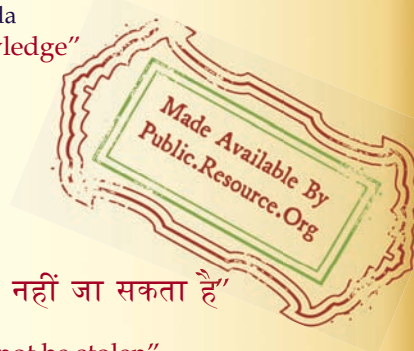"Step Out From the Old to the New"

IS 6200-3 (2003): Statistical Tests of Significance, Part 3: Tests for Normality [MSD 3: Statistical Methods for Quality and Reliability]

"ज्ञान से एक नये भारत का निर्माण"
Satyanarayan Gangaram Pitroda
"Invent a New India Using Knowledge"

"ज्ञान एक ऐसा खजाना है जो कभी चुराया नहीं जा सकता है"
Bhartṛhari—Nītiśatakam
"Knowledge is such a treasure which cannot be stolen"

BLANK PAGE

भारतीय मानक

सार्थकता के लिए सांख्यिकीय परीक्षण

भाग 3 नार्मलिटी के परीक्षण

(दूसरा पुनरीक्षण)

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 3 TESTS FOR NORMALITY

(Second Revision)

ICS 03.120.30

BUREAU OF INDIAN STANDARDS
MANAK BHAVAN, 9 BAHADUR SHAH ZAFAR MARG
NEW DELHI 110002

FOREWORD

This Indian Standard (Part 3) (Second Revision) was adopted by the Bureau of Indian Standards, after the draft finalized by Statistical Methods for Quality and Reliability Sectional Committee had been approved by the Management and Systems Division Council.

This standard was originally published in 1971 and covered the industrial applications of three main tests of significance, namely, t-test, F-test and χ^2-test. It was then revised in 1977 into four parts to include tests for normality and also some non-parametric tests, which have wide application in industry.

This revision of the standard has been undertaken to

a) include basic concepts of testing of hypothesis,

b) exclude the graphical test for normality as it is not a method of test of significance. Moreover, for this purpose, a separate IS 12348 : 1998 'Use of probability papers,' already exists, and

c) incorporate many technical and editorial corrections.

Some of the statistical tests of significance discussed in Parts 1 and 2 of this standard are based on the assumption that the variable under consideration follows normal distribution. But it is not always necessary to apply the test for normality whenever the statistical tests based on normality assumptions are used. For example, this test may not be applied in case of robust tests because the test results are little affected even if the normality assumption does not hold good. Student's t-test is one such test where the test of normality is not very important. However, where the tests are not robust and there is doubt about the normality assumption, the use of this standard may be very useful or even necessary.

The use of normality tests is not limited to the cases mentioned above. Normality tests are also used to check the validity of normal distribution for statistical purposes or, where a generating process produces items where parameters are distributed in accordance with normal distribution.

In addition to this Part 3, IS 6200 has following three parts:

Part 1 Normal, t- and F-tests

Part 2 χ^2-test

Part 4 Non-parametric tests

The composition of the Committee responsible for the formulation of this standard is given in Annex E.

Indian Standard

STATISTICAL TESTS OF SIGNIFICANCE

PART 3 TESTS FOR NORMALITY

(Second Revision)

1 SCOPE

This standard (Part 3) gives the following tests for normality:

a) χ^2-test,

b) Normal test based on skewness and kurtosis,

c) Shapiro-Wilk test, and

d) D'Agostino test.

2 REFERENCES

The following standards contain provisions, which through reference in this text constitute provisions of this standard. At the time of publication, the editions indicated were valid. All standards are subject to revision and parties to agreements based on this standard are encouraged to investigate the possibility of applying the most recent editions of the standards indicated below:

IS No.	Title
6200	Statistical tests of significance:
(Part 1) : 2003	Normal, *t*- and *F*-tests
(Part 2) : 2003	χ^2-test
7920	Statistical vocabulary and symbols:
(Part 1) : 1994	Probability and general statistical terms (*second revision*)
(Part 2) : 1994	Statistical quality control (*second revision*)
9300 (Part 2) : 1979	Statistical models for industrial applications: Part 2 Continuous models
12348 : 1988	Use of probability papers

3 TERMINOLOGY

For the purpose of this standard, the definitions as given in IS 7920 (Part 1) and IS 7920 (Part 2) shall apply.

4 BASIC CONCEPTS

4.1 Statistical tests of significance are important tools in decision-making. They are extremely useful in finding out whether, in the case of one population, the mean value differs significantly from certain specified value or whether, in the case of two populations, the mean values differ significantly from each other. Thus, it may be desirable to find out whether a new germicide is more effective in treating a certain type of infection than a standard germicide, whether a new method of sealing light bulbs will increase their life or whether one method of preserving foods is better than another in so far as the retention of vitamins is concerned. In such cases, it would be necessary to examine whether the mean values can be deemed as same or different. There may also be cases where it may be worthwhile to find out whether one inspector is more consistent than another or whether a new source of raw material has resulted in a change in the variability of the output, or whether the temperature of the bath in which the cocoons are cooked affects the uniformity of the quality of silk. In these cases it will be necessary to determine whether the variances are the same or not.

4.2 Formulation of Hypotheses

For taking a decision using statistical tests of significance, the first step is to form the hypotheses, namely, Null Hypothesis (H_0) and Alternative Hypothesis (H_1).

4.2.1 *Null Hypothesis (H_0)*

The procedure commonly used is to first set up a null hypothesis regarding equivalence (no difference). The question on which the decision is called for, by applying the tests of significance, is translated in terms of null hypothesis in such a way that this null hypothesis would likely to be rejected if there is enough evidence against it as seen from the data in the sample. For example, a null hypothesis will be that the data follow a normal distribution.

4.2.2 *Alternative Hypothesis (H_1)*

Alternative hypothesis is a hypothesis that will be preferred in case the null hypothesis is not true.

4.3 Level of Significance

4.3.1 There are two kinds of errors involved in taking the decision based on the tests of significance, namely,

a) *Type I Error* — Error in deciding that a significant difference exists when there is no real difference.

b) *Type II Error* — Error in deciding that no difference exists when there is a real difference.

4.3.2 Type I error and Type II error are also called Error of the first kind and Error of the second kind respectively. This process of decision-making is

described in Table 1.

Table 1 Process of Decision-Making

(*Clause* 4.3.2)

	H_0 True	H_1 True
Reject H_0	Type I error	Correct decision
Accept H_0	Correct decision	Type II error

4.3.3 Based on the distribution of test statistics used, it is possible to work out the probability of committing Type I error. The probability of committing Type I error is called level of significance (α). The probability of committing Type II error is called level of significance (β). It is not possible to minimize both these probabilities (risk) at the same time. Hence, assigning to it a chosen level of probability controls one of the risks, usually of the first kind. Generally the value for level of significance is chosen as 0.05 or 0.01, that is, 5 percent or 1 percent. This implies confidence level of 95 percent or 99 percent respectively.

4.4 The decision-making procedure involves the comparison of the calculated value of the statistic with the tabulated value. If the calculated value is greater than or equal to the tabulated value of the statistic, then H_0 is rejected, thereby accepting H_1; otherwise H_0 is not rejected. For practical purpose, H_0 not rejected is taken as if it is accepted.

5 CHI-SQUARE (χ^2) TEST FOR NORMALITY

5.1 Chi-square test may also be used for testing the normality. Let there be n random sample observations.

The frequency distribution is obtained from these observations. The expected frequencies for each class interval under the null hypothesis of normality is then obtained. The procedure for obtaining the expected frequencies is illustrated with an example given in **5.2** [*see* IS 9300 (Part 2)].

5.2 Example

The mean and standard deviation of 150 observations of leakage current (μA) taken on 150 electric irons are 37.6 μA and 12.3 μA, respectively. These values are given in Table 2 in the form of frequency distribution. Test whether the data follow normal distribution.

5.2.1 Here the null hypothesis is H_0 : The data follow normal distribution, to be tested against the alternative hypothesis H_1: The data do not follow normal distribution.

5.2.2 The area of the left of upper limit of each class interval is calculated with the help of standard normal tables [*see* IS 9300 (Part 2)] as given in Table 3.

5.2.3 Applying χ^2-test for goodness of fit [*see* IS 6200 (Part 2)], that is,

$$\chi^2 = \sum_{i=1}^{10} \frac{(o_i - e_i)^2}{e_i}$$

The calculated value of χ^2 comes out as 3.69. The tabulated value for 7 degrees of freedom and at 5 percent level of significance is 14.07 is given in Annex A. Since the calculated value is less than the tabulated value, the null hypothesis that the data follow normal distribution is not rejected.

Table 2 Frequency Distribution of Leakage Current

(*Clause* 5.2)

Class-Interval (1)	Tally Mark (2)	Frequency (3)
10.5 – 15.5	⦀⦀ /	6
15.5 – 20.5	⦀⦀ ///	8
20.5 – 25.5	⦀⦀ ⦀⦀ //	12
25.5 – 30.5	⦀⦀ ⦀⦀ ⦀⦀ /	16
30.5 – 35.5	⦀⦀ ⦀⦀ ⦀⦀ ⦀⦀ /	21
35.5 – 40.5	⦀⦀ ⦀⦀ ⦀⦀ ⦀⦀ ⦀⦀ ⦀⦀	30
40.5 – 45.5	⦀⦀ ⦀⦀ ⦀⦀ ///	18
45.5 – 50.5	⦀⦀ ⦀⦀ ⦀⦀	15
50.5 – 55.5	⦀⦀ ⦀⦀	10
55.5 – 60.5	⦀⦀ ////	9
60.5 – 65.5	⦀⦀	5
Total		150

Table 3 Area Under Normal Curve to Left of Upper Limit of Each Class Interval

(Clause 5.2.2)

Upper Limit of Class Interval	$z = \dfrac{x-37.6}{12.3}$	$F(x)$ – Area to Left of z	Area within Interval	Expected Frequency (e_i) = (4) × 150	Observed Frequency (o_i)
(1)	(2)	(3)	(4)	(5)	(6)
15.5	− 1.80	0.035 9	0.035 9	5.4	6
20.5	− 1.39	0.082 3	0.046 4	7.0	8
25.5	− 0.98	0.163 5	0.081 2	12.2	12
30.5	− 0.58	0.281 0	0.117 5	17.6	16
35.5	− 0.17	0.432 5	0.151 5	22.7	21
40.5	0.24	0.594 8	0.162 3	24.3	30
45.5	0.64	0.738 9	0.144 1	21.6	18
50.5	1.05	0.853 1	0.114 2	17.1	15
55.5	1.46	0.927 9	0.074 8	11.2	10
60.5	1.86	0.968 6	0.040 7	6.1 ⎫	9 ⎫
65.5	2.27	0.988 4	0.019 8	3.0 ⎬ Pooled	5 ⎬ Pooled
		1.000 0	0.011 6	1.8 ⎭	0 ⎭

6 NORMALITY TEST BASED ON SKEWNESS AND KURTOSIS

6.1 This test is based on the fact that if the sample is drawn from a normal population then the skewness ($\beta_3 = 0$) and kurtosis ($\beta_3 = 0$) and kurtosis will be very near to zero and three respectively. For this purpose, the values of skewness and kurtosis are calculated from the observation using the following relations:

Measurement of Skewness $(b_1) = \dfrac{m_3^{\,2}}{m_2^{\,3}} = \dfrac{\left[1/n \sum\limits_{i=1}^{n} (x_i - \bar{x})^3\right]^2}{\left[1/n \sum\limits_{i=1}^{n} (x_i - \bar{x})^2\right]^3}$

Measurement of Skewness $(b_2) = \dfrac{m_4}{m_2^{\,2}} = \dfrac{\left[1/n \sum\limits_{i=1}^{n} (x_i - \bar{x})^4\right]}{\left[1/n \sum\limits_{i=1}^{n} (x_i - \bar{x})^2\right]^2}$

6.2 The calculated values of b_1 and b_2 so obtained are tested for $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \gamma 0$ and $H_0 : \beta_2 = 3$ against $H_1 : \beta_2 \gamma 3$ respectively.

6.2.1 For large samples, the statistics,

$\sqrt{nb_1/6} \sim N(0,1)$ & $\{|b_2 - 3| \sqrt{n}\}/\sqrt{24} \sim N(0,1)$.

6.2.2 The null hypothesis $H_0 : \beta_1 = 0$ is rejected if the value of $\sqrt{nb_1/6}$ is greater than the tabulated z value at a given level of significance [*see* IS 6200 (Part 1)], otherwise not. Similarly, the null hypothesis $H_0 : \beta_2 = 3$ is rejected if the value of $\{|b_2 - 3| \sqrt{n}\}/\sqrt{24}$ is greater than the tabulated z value at a given level of significance [*see* IS 6200 (Part 1)], otherwise not.

6.2.3 If both the above null hypotheses, namely,

$H_0 : \beta_1 = 0$ and $H_0 : \beta_2 = 3$ are not rejected, then the data may be assumed to come from normal distribution.

6.3 Example

The mass (in kg) of 50 steel tubes of 100 mm nominal bore are given in Table 4. Test whether the data follow normal distribution.

6.3.1 The calculations are as follows:

Mean $(\bar{x}) = 1/50 \sum\limits_{i=1}^{50} x_i = 10.14$

$m_2 = 1/50 \sum\limits_{i=1}^{50} (x_i - 10.14)^2 = 0.202\ 0$

$m_3 = 1/50 \sum\limits_{i=1}^{50} (x_i - 10.14)^3 = 0.025\ 6$

$m_4 = 1/50 \sum\limits_{i=1}^{50} (x_i - 10.14)^4 = 0.083\ 9$

$b_1 = m_3^{\,2} / m_2^{\,3} = 0.079\ 5$

$b_2 = m_4 / m_2^{\,2} = 2.056\ 2$

$\sqrt{\dfrac{nb_1}{6}} = \sqrt{\dfrac{50 \times 0.079\ 5}{6}} = 0.81$, and

$\dfrac{|b_2 - 3| \sqrt{n}}{\sqrt{24}} = \dfrac{|2.056\ 2 - 3| \ \sqrt{50}}{\sqrt{24}} = 1.36$

Since the calculated values of $\sqrt{\dfrac{nb_1}{6}}$ and $\dfrac{|b_2 - 3| \sqrt{n}}{\sqrt{24}}$ are less than 1.96, the null hypotheses, namely, $H_0 : \beta_1 = 0$ and $H_0 : \beta_2 = 3$ are not rejected at 5 percent level of significance, hence the data are assumed to follow normal distribution.

3

Table 4 Mass (kg) of Steel Tubes

(Clause 6.3)

10.35	9.54	10.18	10.42	10.15
10.27	10.08	10.26	10.28	9.60
9.65	10.05	9.23	10.32	10.25
10.32	10.10	9.36	10.55	10.78
10.25	10.15	9.48	10.65	10.35
10.85	10.60	10.15	9.86	10.45
9.72	10.40	10.36	9.45	10.94
9.75	10.72	10.48	9.82	10.63
10.25	9.70	10.79	9.40	10.47
9.35	9.80	10.82	9.50	10.15

7 SHAPIRO-WILK TEST ($3 \leq n \leq 50$)

7.1 Let independent observations be arranged in non-decreasing sequence and be designated by $x_{(1)}$, $x_{(2)}$, ..., $x_{(n)}$. The value of S is calculated by the following relation:

$$S = \Sigma_k \, a_k \, [\, x_{(n+1-k)} - x_{(k)}]$$

where $k = 1, 2 \ldots \ldots n/2$ when n is even

$\qquad = 1, 2 \ldots \ldots (n-1)/2$ when n is odd.

The values of coefficient 'a_k' are given in Annex B for different values of n and k.

7.2 If some observations are equal, the ordered series is enumerated by repeating the equal values as many times as they occur in the original series.

7.3 The test statistic W is computed as:

$$W = \frac{S^2}{n \, m_2}$$

7.3.1 The null hypothesis, that is H_0 : The data follows a normal distribution, is accepted at desired level of significance (5 percent or 1 percent), if the calculated value of W is greater than the tabulated value for given n. The tabulated critical value of W are given in Annex C for 5 percent and 1 percent levels of significance.

7.4 Example

Table 5 gives the annual rainfall collected at a meteorological station. For statistical reasons it is desired to investigate whether the annual rainfall follow normal distribution.

Table 5 Rainfall in cm

(Clause 7.4)

2.82	8.47	0.22	0.92	4.39	6.56
0.80	2.43	3.45	6.49	0.54	3.88
9.94	7.44	2.68	2.07	8.47	4.97
0.97	6.85	3.91	9.02	8.86	1.79
2.81	1.62	2.97	1.32	2.51	7.20

7.4.1 Table 6 gives the values of rainfall when arranged in non-decreasing order.

7.4.2 In order to make the calculations easy, the procedure given in **7.4.2.1** is adopted.

7.4.2.1 If the number of observations is even write the first $n/2$ values of $x_{(k)}$ in a column for $x_{(k)}$ (*see* col 2 of Table 6). The last $n/2$ values of $x_{(k)}$ are entered from bottom in the next column for $x_{(n+1-k)}$, that is, $x_{(n/2+1)}$th value will be at the bottom and $x_{(n)}$th value in the beginning of the next column for $x_{(n+1-k)}$ (*see* col 3 of Table 6).

7.4.2.2 If the number of observations is odd, write the first $(n-1)/2$ values of $x_{(k)}$ in a column for $x_{(k)}$, delete the middle value, and enter the last $(n-1)/2$ values of $x_{(k)}$ from bottom of next column for $x_{(n+1-k)}$.

7.4.3

$$\bar{x} = (1/30) \sum_{i=1}^{30} x_i = 126.37/30 = 4.21$$

$$nm_2 = \sum_{i=1}^{30} (x_i - \bar{x})^2$$

$$= 254.00$$

$$S = \Sigma \, a_k \, [x_{(n+1-k)} - x_{(k)}]$$

$$= 0.425\,4 \times 9.72 + 0.294\,4 \times 8.48 + \ldots . + 0.007\,6 \times 0.48$$

$$\approx 15.21$$

$$W = \frac{S^2}{n \, m_2} = \frac{231.34}{254.00} = 0.911$$

7.5 Since the tabulated value of W at 5 percent level of significance corresponding to $n = 30$ is 0.927 which exceeds the value of W obtained above, the null hypothesis, H_0 : The data follow normality, is rejected at 5 percent level of significance.

8 D'AGOSTINO TEST ($50 < n \leq 1\,000$)

8.1 When the number of observations is more than 50,

4

Table 6 Rainfall in Non-decreasing Order

(*Clause* 7.4.1)

k	$x_{(k)}$	$x_{(n+1-k)}$	$x_{(n+1-k)} - x_{(k)}$	a_k	$a_k[x_{(n+1-k)} - x_{(k)}]$ = (4) × (5)
(1)	(2)	(3)	(4)	(5)	(6)
1	0.22	9.94	9.72	0.425 4	4.134 9
2	0.54	9.02	8.48	0.294 4	2.496 5
3	0.80	8.86	8.06	0.248 7	2.004 5
4	0.92	8.47	7.55	0.214 8	1.621 7
5	0.97	8.47	7.50	0.187 0	1.402 6
6	1.32	7.44	6.12	0.163 0	0.997 6
7	1.62	7.20	5.58	0.141 5	0.789 6
8	1.79	6.85	5.06	0.121 9	0.616 8
9	2.07	6.56	4.49	0.103 6	0.465 2
10	2.43	6.49	4.06	0.086 2	0.350 0
11	2.51	4.97	2.46	0.069 7	0.171 5
12	2.68	4.39	1.71	0.053 7	0.091 8
13	2.81	3.91	1.10	0.038 1	0.041 9
14	2.82	3.88	1.06	0.022 7	0.024 1
15	2.97	3.45	0.48	0.007 6	0.003 6
					$S = 15.212\ 2$

Shapiro-Wilk test cannot be applied. This test is also based on ordered statistics and is applicable only when the number of observations is more than 50.

8.2 Let independent observations be arranged in non-decreasing sequence and is designated by $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$. The values of S and D may be calculated as follows:

$$S = \Sigma_k a_k [x_{(n+1-k)} - x_{(k)}]$$

where $k = 1, \ldots \ldots n/2$ if n is even

$k = 1, \ldots, (n-1)/2$, if n is odd

$a_k = (n+1)/2 - k$, and

$$D = \frac{S}{n^2 \sqrt{m_2}}$$

8.3 The test statistic is given by :

$$\gamma = \sqrt{n}\,(D - 0.282\ 1)/0.03$$

8.4 Under the null hypothesis, H_0 : The data follows normal distribution, the value of γ is zero. The critical values for $n = 50\,(10)\,100\,(50)\,1\,000$ at desired level of significance are given in Annex D. If the calculated

value of γ, for a given n and level of significance, lie between lower and upper value, then the null hypothesis of normality is not rejected.

8.5 Example

The test results of the Hardness (in HB) of 54 tools are given in non-decreasing order in Table 7. It is to be tested whether the data comes from a normal distribution.

8.6 Calculations

$$m_2 = (1/54)\, \Sigma\, (x_i - \bar{x})^2 = 11\,685.62$$

Therefore, $D = \dfrac{S}{n^2 \sqrt{m_2}} = \dfrac{85\,563}{2\,916 \times 108.1} = 0.271\ 4$

Hence $\gamma = \dfrac{\sqrt{54}\,(0.217\ 4 - 0.282\ 1)}{0.03}$

$$= -2.61$$

8.7 The lower and upper critical values of γ from Annex D at 5 percent level of significance, when interpolated, comes out as −2.71 and 1.10 respectively. Since the calculated value of γ lies between −2.71 and 1.10, the null hypothesis of normality is not rejected at 5 percent level of significance.

Table 7 Hardness of Tools (HB)

(Clause 8.5)

k	$x_{(k)}$	$x_{(n+1-k)}$	$\|x_{(n+1-k)} - x_{(k)}\|$	a_k	$a_k[x_{(n+1-k)} - x_{(k)}]$ = (4) × (5)
(1)	(2)	(3)	(4)	(5)	(6)
1	520	1 074	554	26.5	14 681.0
2	556	1 056	500	25.5	12 750.0
3	561	963	402	24.5	9 849.0
4	616	952	336	23.5	7 896.0
5	635	926	291	22.5	6 547.5
6	669	922	253	21.5	5 439.5
7	686	904	218	20.5	4 469.0
8	692	900	208	19.5	4 056.0
9	704	889	185	18.5	3 422.5
10	707	879	172	17.5	3 010.0
11	711	873	162	16.5	2 673.0
12	713	862	149	15.5	2 309.5
13	714	851	137	14.5	1 986.5
14	719	837	118	13.5	1 539.0
15	727	834	107	12.5	1 337.5
16	735	826	91	11.5	1 046.5
17	740	822	82	10.5	861.0
18	744	821	77	9.5	731.5
19	745	794	49	8.5	416.5
20	750	791	41	7.5	307.5
21	776	786	10	6.5	65.0
22	777	786	9	5.5	49.5
23	777	785	8	4.5	36.0
24	780	785	5	3.5	17.5
25	780	784	4	2.5	10.0
26	781	782	1	1.5	1.5
27	781	782	1	0.5	0.5
					S = 85 563.0

ANNEX A
(*Clause* 5.2.3)
CRITICAL VALUES OF χ^2 – DISTRIBUTION

Degree of Freedom	Significance Level		Degree of Freedom	Significance Level	
	0.05	0.01		0.05	0.01
(1)	(2)	(3)	(1)	(2)	(3)
1	3.84	6.64	20	31.41	37.56
2	5.99	9.21	21	32.67	38.93
3	7.82	11.34	22	33.92	40.29
4	9.49	13.28	23	35.17	41.64
5	11.07	15.09	24	36.42	42.98
6	12.59	16.81	25	37.65	44.31
7	14.07	18.48	26	68.89	45.64
8	15.51	20.09	27	40.11	46.96
9	16.92	21.67	28	41.34	48.28
10	18.31	23.21	29	42.56	49.59
11	19.68	24.73	30	43.77	50.89
12	21.03	26.22	40	55.75	63.69
13	22.36	27.69	50	67.50	76.15
14	23.69	29.14	60	79.18	88.38
15	25.00	30.58	70	90.53	100.42
16	26.30	32.00	80	101.88	112.33
17	27.59	33.41	90	113.14	124.12
18	28.87	34.81	100	124.34	135.81
19	30.14	36.19			

ANNEX B

(*Clause* 7.1)

COEFFICIENTS a_k NECESSARY TO CALCULATE 'S'

n \ k	2	3	4	5	6	7	8	9	10
1	0.707 1	0.707 1	0.687 2	0.643 1	0.643 1	0.623 3	0.605 2	0.588 8	0.573 9
2	–	.000 0	.241 3	.280 6	.280 6	.303 1	.316 4	.324 4	.329 1
3	–	–	.000 0	.087 5	.087 5	.140 1	.174 3	.197 6	.214 1
4	–	–	–	–	–	.000 0	.056 1	.094 7	.122 4
5	–	–	–	–	–	–	–	.000 1	.039 9

	11	12	13	14	15	16	17	18	19	20
1	0.580 1	0.547 5	0.535 9	0.525 1	0.515 0	0.505 6	0.496 8	0.488 6	0.480 8	0.473 4
2	.331 5	.332 5	.332 5	.331 8	.330 6	.329 0	.327 3	.325 3	.323 2	.321 1
3	.226 0	.234 7	.241 2	.246 0	.249 5	.252 1	.254 0	.255 3	.256 1	.256 5
4	.142 9	.158 6	.170 7	.180 2	.187 8	.193 9	.198 8	.202 7	.205 9	.208 5
5	.000 5	.092 2	.109 9	.124 0	.135 3	.144 7	.152 4	.158 7	.164 1	.168 6
6	0.000 0	0.000 0	0.053 9	0.072 7	0.088 0	0.100 5	0.110 9	0.119 7	0.127 1	0.133 4
7	–	–	.000 00	.240 0	.043 3	.059 3	.072 5	.083 7	.093 2	.101 3
8	–	–	–	–	.000 0	.019 6	.035 9	.049 6	.061 2	.071 1
9	–	–	–	–	–	–	.000 0	.016 3	.030 3	.042 2
10	–	–	–	–	–	–	–	–	.000 0	.014 0

	21	22	23	24	25	26	27	28	29	30
1	0.464 3	0.459 0	0.454 2	0.449 3	0.445 0	0.440 7	0.436 6	0.432 8	0.429 1	0.425 4
2	.318 5	.315 6	.312 6	.309 8	.306 9	.304 3	.301 8	.299 2	.296 8	.294 4
3	.257 8	.257 1	.256 3	.255 4	.254 3	.253 3	.252 2	.251 0	.249 9	.248 7
4	.211 9	.213 1	.213 9	.214 5	.214 8	.215 1	.215 2	.215 1	.215 0	.214 8
5	.173 6	.176 4	.178 7	.180 7	.182 2	.183 6	.184 8	.185 7	.186 4	.187 0
6	0.139 9	0.144 3	0.148 0	0.151 2	0.153 9	0.156 3	0.158 4	0.160 1	0.161 6	0.163 0
7	.109 2	.115 0	.120 1	.124 5	.128 3	.131 6	.134 6	.137 2	.139 5	.141 5
8	.080 4	.087 8	.094 1	.099 7	.104 6	.108 9	.112 8	.116 2	.119 2	.121 9
9	.053 0	.061 8	.069 6	.076 4	.082 3	.087 6	.092 3	.096 5	.100 2	.103 6
10	.026 3	.036 8	.045 9	.053 9	.061 0	.067 2	.072 8	.077 8	.082 2	.086 2
11	0.000 0	0.012 2	0.022 8	0.032 1	0.040 3	0.047 6	0.054 0	0.059 8	0.065 0	0.069 7
12	–	–	.000 0	.010 7	.020 0	.028 4	.035 8	.042 4	.048 3	.053 7
13	–	–	–	.000 0		.009 4	.017 8	.025 3	.032 0	.038 1
14	–	–	–	–	–	–	.000 0	.008 4	.015 9	.022 7
15	–	–	–	–	–	–	–	–	.000 0	.007 6

	31	32	33	34	35	36	37	38	39	40
1	0.422 0	0.418 8	0.415 6	0.412 7	0.409 6	0.406 8	0.404 0	0.401 5	0.398 9	0.396 4
2	.292 1	.289 8	.287 6	.285 4	.283 4	.281 3	.279 4	.277 4	.275 5	.273 7
3	.247 5	.246 3	.245 1	.243 9	.242 7	.241 5	.240 3	.239 2	.238 0	.236 8
4	.214 5	.214 1	.213 7	.213 2	.212 7	.212 7	.211 6	.211 0	.210 4	.209 8
5	.187 4	.187 8	.188 0	.188 2	.188 3	.188 3	.188 3	.188 1	.188 0	.187 8
6	.164 1	.165 1	.166 0	.166 7	.167 3	.167 8	.168 3	.168 6	.168 9	.169 1
7	.143 3	.144 9	.146 3	.147 5	.148 7	.149 6	.150 5	.151 3	.152 0	.152 6
8	.124 3	.126 5	.128 4	.130 1	.131 7	.133 1	.134 4	.135 6	.136 6	.137 6

ANNEX B (*Concluded*)

k \ n	31	32	33	34	35	36	37	38	39	40
9	.106 6	.109 3	.111 8	.114 0	.116 0	.117 9	.119 6	.121 1	.122 5	.123 7
10	.089 9	.093 1	.096 1	.098 8	.101 3	.103 6	.105 6	.107 5	.109 2	.110 8
11	0.073 9	0.077 7	0.081 2	0.084 4	0.087 3	0.090 0	0.092 4	0.094 7	0.096 7	0.098 6
12	.058 5	.062 9	.066 9	.000 6	.073 9	.077 0	.079 8	.082 4	.084 8	.087 0
13	.043 5	.048 5	.053 0	.057 2	.061 0	.064 5	.067 7	.070 6	.073 3	.075 9
14	.028 9	.034 4	.039 5	.044 1	.048 4	.052 3	.055 9	.059 2	.062 2	.065 1
15	.014 4	.020 6	.026 2	.031 4	.036 1	.040 4	.044 4	.048 1	.051 5	.054 6
16	.000 0	.006 8	.013 1	.018 7	.023 9	.028 7	.033 1	.037 2	.040 9	.044 4
17	–	–	.000 0	.006 2	.011 9	.017 2	.022 0	.026 4	.030 5	.034 3
18	–	–	–	–	.000 0	.005 7	.011 1	.015 8	.020 3	.024 4
19	–	–	–	–	–	–	.000 0	.005 3	.010 1	.014 6
20	–	–	–	–	–	–	–	–	.000 0	.004 9

k \ n	41	42	43	44	45	46	47	48	49	50
1	0.384 0	0.391 7	0.389 4	0.387 2	0.3850	0.3830	0.380 8	0.378 9	0.377 0	0.375 1
2	.2719	.2701	.2684	.2667	.2651	.2635	.2620	.2604	.2589	.2574
3	.2357	.2345	.2334	.2323	.2313	.2302	.2291	.2281	.2271	.2260
4	.209 1	.208 5	.207 8	.207 2	.2065	.205 8	.205 2	.204 5	.203 8	.203 2
5	.187 6	.187 4	.187 1	.186 8	.1865	.186 2	.185 9	.185 5	.185 1	.184 7
6	0.169 3	0.169 4	0.169 5	0.169 5	0.169 5	0.169 5	0.169 5	0.169 3	0.169 2	0.169 1
7	.153 1	.153 5	.153 9	.154 2	.154 5	.154 8	.1550	.155 1	.155 6	.155 4
8	.138 4	.139 2	.139 8	.140 5	.141 0	.141 5	.1420	.142 3	.142 7	.143 0
9	.124 9	.125 9	.126 9	.127 8	.128 6	.129 3	.130 0	.130 6	.131 2	.131 7
10	.112 3	.113 6	.114 9	.116 0	.117 0	.118 0	.118 9	.119 7	.120 5	.121 2
11	0.100 4	0.102 0	0.103 5	0.104 9	0.106 2	0.107 3	0.108 5	0.109 5	0.110 5	0.111 5
12	.089 1	.090 9	.092 7	.094 3	.095 9	.097 2	.098 6	.099 8	.101 0	.102 0
13	.078 2	.080 4	.082 4	.094 2	.086 0	.087 6	.089 2	.090 6	.091 9	.093 2
14	.067 7	.070 1	.072 4	.075 4	.076 5	.078 6	.080 1	.081 7	.083 2	.084 6
15	.057 5	.060 2	.062 8	.065 1	.067 3	.069 4	.071 3	.073 1	.074 8	.076 4
16	0.047 6	0.050 6	0.053 4	0.056 0	0.058 4	0.060 7	0.062 8	0.064 8	0.066 7	0.068 5
17	.037 9	.041 1	.044 2	.047 1	.049 7	.052 2	.054 6	.056 8	.058 8	.060 8
18	.028 3	.031 8	.035 2	.038 3	.041 2	.043 9	.043 5	.048 9	.051 1	.053 2
19	.018 8	.022 7	.026 3	.029 6	.032 8	.035 7	.038 5	.041 1	.043 6	.045 9
20	.009 4	.013 6	.017 5	.021 1	.024 5	.027 7	.030 7	.033 5	.036 1	.038 6
21	0.000 0	0.004 5	0.008 7	0.012 6	0.016 3	0.019 7	0.022 9	0.025 9	0.028 8	0.031 4
22	–	–	.000	0.004 2	.011 8	.011 8	.015 3	.018 5	.021 5	.024 4
23	–	–	–	–	.000 0	.003 9	.007 6	.011 1	.014 3	.017 4
24	–	–	–	–	–	–	.000 0	.003 7	.007 1	.010 4
25	–	–	–	–	–	–	–	–	.000 0	.003 5

ANNEX C

(*Clause* 7.3.1)

SHAPIRO-WILK TEST — CRITICAL VALUES OF *W*

n	Level of Significance		*n*	Level of Significance	
	5 percent	1 percent		5 percent	1 percent
3	0.767	0.753	26	0.920	0.891
4	.648	.687	27	.923	.894
5	.762	.686	28	.924	.896
			29	.926	.898
6	0.788	0.713	30	.927	.900
7	.803	.730			
8	.818	.749	31	0.929	0.902
9	.829	.764	32	.930	.904
10	.842	.781	33	.931	.906
			34	.933	.908
11	0.850	0.792	35	.934	.910
12	.859	.805			
13	.866	.814	36	0.935	0.912
14	.874	.825	37	.936	.914
15	.881	.835	38	.938	.916
			39	.939	.917
16	0.887	0.844	40	.940	.919
17	.892	.851			
18	.897	.858	41	0.941	0.920
19	.901	.863	42	.942	.922
20	.905	.868	43	.943	.923
			44	.944	.924
21	0.908	0.873	45	.945	.926
22	.911	.878			
23	.914	.881	46	0.945	0.927
24	.916	.884	47	.946	.928
25	.918	.888	48	.947	.929
			49	.947	.929
			50	.947	.930

ANNEX D

(Clauses 8.4 and 8.7)

D'AGOSTINO TEST — CRITICAL VALUES OF γ

n	Level of Significance			
	5 Percent		1 Percent	
	Lower	Upper	Lower	Upper
50	−2.74	1.06	−3.91	1.24
60	−2.68	1.13	−3.81	1.34
70	−2.64	1.19	−3.73	1.42
80	−2.60	1.24	−3.67	1.48
90	−2.57	1.28	−3.61	1.54
100	−2.54	1.31	−3.57	1.59
150	−2.452	1.423	−3.409	1.746
200	−2.391	1.496	−3.302	1.853
250	−2.348	1.545	−3.227	1.927
300	−2.316	1.528	−3.172	1.983
350	−2.291	1.610	−3.129	2.026
400	−2.270	1.633	−3.094	2.061
450	−2.253	1.652	−3.064	2.090
500	−2.239	1.668	−3.040	2.114
550	−2.226	1.682	−3.019	2.136
600	−2.215	1.694	−3.000	2.154
650	−2.206	1.704	−2.984	2.171
700	−2.197	1.714	−2.969	2.185
750	−2.189	1.722	−2.956	2.199
800	−2.182	1.730	−2.944	2.211
850	−2.176	1.737	−2.933	2.221
900	−2.170	1.743	−2.923	2.231
950	−2.164	1.749	−2.914	2.241
1 000	−2.150	1.754	−2.906	2.249

ANNEX E

(*Foreword*)

COMMITTEE COMPOSITION

Statistical Methods for Quality and Reliability Sectional Committee, MSD 3

Organization	*Representative(s)*
Kolkata University, Kolkata	PROF S. P. MUKHERJEE (**Chairman**)
Bharat Heavy Electricals Limited, Hyderabad	SHRI S. N. JHA
	SHRI A. V. KRISHNAN (*Alternate*)
Continental Devices India Ltd, New Delhi	DR NAVIN KAPUR
	SHRI VIPUL GUPTA (*Alternate*)
Directorate General of Quality Assurance, New Delhi	SHRI S. K. SRIVASTVA
	LT COL P. VIJAYAN (*Alternate*)
Laser Science and Technology Centre, DRDO, New Delhi	DR ASHOK KUMAR
Escorts Limited, Faridabad	SHRI C. S. V. NARENDRA
HMT Ltd, R & D Centre, Bangalore	SHRI K. VIJAYAMMA
Indian Agricultural Statistics Research Institute, New Delhi	DR S. D. SHARMA
	DR A. K. SRIVASTAVA (*Alternate*)
Indian Association for Productivity, Quality & Reliability, Kolkata	DR B. DAS
Indian Institute of Management, Lucknow	PROF S. CHAKRABORTY
Indian Statistical Institute, Kolkata	PROF S. R. MOHAN
	PROF ARVIND SETH (*Alternate*)
National Institution for Quality and Reliability, New Delhi	SHRI Y. K. BHAT
	SHRI G. W. DATEY (*Alternate*)
POWERGRID Corporation of India Ltd, New Delhi	DR S. K. AGARWAL
	SHRI D. CHAKRABORTY (*Alternate*)
SRF Limited, Chennai	SHRI A. SANJEEVA RAO
	SHRI C. DESIGAN (*Alternate*)
Standardization, Testing and Quality Certification Directorate, New Delhi	SHRI S. K. KIMOTHI
	SHRI P. N. SRIKANTH (*Alternate*)
Tata Engineering and Locomotive Co Ltd, Jamshedpur	SHRI S. KUMAR
	SHRI SHANTI SARUP (*Alternate*)
University of Delhi, Delhi	PROF M. C. AGRAWAL
In personal capacity (*B-109, Malviya Nagar, New Delhi 110017*)	PROF A. N. NANKANA
In personal capacity (*20/1, Krishna Nagar, Safdarjung Enclave, New Delhi 110029*)	SHRI D. R. SEN
BIS Directorate General	SHRI P. K. GAMBHIR, Director and Head (MSD)
	[Representing Director General (*Ex-officio*)]

Member Secretary
SHRI LALIT KUMAR MEHTA
Joint Director (MSD), BIS

Basic Statistical Methods Subcommittee, MSD 3:1

Kolkata University, Kolkata	PROF S. P. MUKHERJEE (**Convener**)
Laser Science and Technology Centre, DRDO, New Delhi	DR ASHOK KUMAR
Indian Agricultural Statistics Research Institute, New Delhi	DR S. D. SHARMA
Indian Association for Productivity, Quality & Reliability, Kolkata	DR B. DAS
	DR A. LAHIRI (*Alternate*)
Indian Institute of Management, Lucknow	PROF. S. CHAKRABORTY
Indian Statistical Institute, New Delhi	PROF S. R. MOHAN
National Institution for Quality and Reliability, New Delhi	SHRI Y. K. BHAT
	SHRI G. W. DATEY (*Alternate*)
POWERGRID Corporation of India Ltd, New Delhi	DR S. K. AGARWAL
Standardization, Testing and Quality Certification Directorate, New Delhi	SHRI S. K. KIMOTHI
Tata Engineering and Locomotive Co Ltd, Pune	SHRI SHANTI SARUP
University College of Medical Sciences, Delhi	DR A. INDRAYAN
University of Delhi, Delhi	PROF M. C. AGRAWAL
In personal capacity (*B-109, Malviya Nagar, New Delhi 110017*)	PROF A. N. NANKANA
In personal capacity (*20/1, Krishna Nagar, Safdarjung Enclave, New Delhi 110029*)	SHRI D. R. SEN

Panel for Basic Methods Including Terminology, MSD 3/P-1

Organization	*Representative(s)*
National Institution for Quality and Reliability, New Delhi	SHRI G. W. DATEY (*Convener*)
Laser Science and Technology Centre, DRDO, New Delhi	DR ASHOK KUMAR
Indian Agricultural Statistics Research Institute, New Delhi	DR. S. D. SHARMA
Indian Statistical Institute, New Delhi	PROF S. R. MOHAN
National Institution for Quality and Reliability, New Delhi	SHRI Y. K. BHAT
POWERGRID Corporation of India Ltd, New Delhi	DR S. K. AGARWAL
In personal capacity *(20/1, Krishna Nagar, Safdarjung Enclave, New Delhi 110029)*	SHRI D. R. SEN

Bureau of Indian Standards

BIS is a statutory institution established under the *Bureau of Indian Standards Act*, 1986 to promote harmonious development of the activities of standardization, marking and quality certification of goods and attending to connected matters in the country.

Copyright

BIS has the copyright of all its publications. No part of these publications may be reproduced in any form without the prior permission in writing of BIS. This does not preclude the free use, in the course of implementing the standard, of necessary details, such as symbols and sizes, type or grade designations. Enquiries relating to copyright be addressed to the Director (Publication), BIS.

Review of Indian Standards

Amendments are issued to standards as the need arises on the basis of comments. Standards are also reviewed periodically; a standard along with amendments is reaffirmed when such review indicates that no changes are needed; if the review indicates that changes are needed, it is taken up for revision. Users of Indian Standards should ascertain that they are in possession of the latest amendments or edition by referring to the latest issue of 'BIS Catalogue' and 'Standards: Monthly Additions'.

This Indian Standard has been developed from Doc: No. MSD 3 (258).

Amendments Issued Since Publication

Amend No.	Date of Issue	Text Affected

BUREAU OF INDIAN STANDARDS

Headquarters:

Manak Bhavan, 9 Bahadur Shah Zafar Marg, New Delhi 110002
Telephones: 2323 0131, 2323 3375, 2323 9402

Telegrams: Manaksanstha
(Common to all offices)

Regional Offices:

Telephone

Central	: Manak Bhavan, 9 Bahadur Shah Zafar Marg NEW DELHI 110002	{ 2323 7617 2323 3841
Eastern	: 1/14 C.I.T. Scheme VII M, V.I.P. Road, Kankurgachi KOLKATA 700054	{ 2337 8499, 2337 8561 2337 8626, 2337 9120
Northern	: SCO 335-336, Sector 34-A, CHANDIGARH 160022	{ 60 3843 60 9285
Southern	: C.I.T. Campus, IV Cross Road, CHENNAI 600113	{ 2254 1216, 2254 1442 2254 2519, 2254 2315
Western	: Manakalaya, E9 MIDC, Marol, Andheri (East) MUMBAI 400093	{ 2832 9295, 2832 7858 2832 7891, 2832 7892
Branches	: AHMEDABAD. BANGALORE. BHOPAL. BHUBANESHWAR. COIMBATORE. FARIDABAD. GHAZIABAD. GUWAHATI. HYDERABAD. JAIPUR. KANPUR. LUCKNOW. NAGPUR. NALAGARH. PATNA. PUNE. RAJKOT. THIRUVANANTHAPURAM. VISAKHAPATNAM.	